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Business cycle filtering of macroeconomic data via a latent business cycle index

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Abstract

We use Markov Chain Monte Carlo methods to augment, via a novel multi-move sampling scheme, a vector autoregressive system with a latent business cycle index that is negative during recessions and positive during expansions. We then sample counterfactual values of the macroeconomic variables in the case where the latent business cycle index is held constant. These counterfactual values represent posterior beliefs about how the economy would have evolved absent business cycle fluctuations. One advantage is that a VAR framework provides model-consistent counterfactual values in the same way that VARs provide model-consistent forecasts, so data series are not filtered in isolation from each other. We apply these methods to estimate the business cycle components of industrial production, consumer price inflation, the federal funds rate and the spread between long-term and short-term interest rates. These decompositions provide an explicitly counterfactual approach to isolating the effects of the business cycle and to deriving empirical business cycle facts.

JEL classifications: **F42, C25, C22**

Key words: **Business cyles, detrending, counterfactual analysis**

1 Introduction

In macroeconomics, transformations of output data to identify or remove the effects of the business cycle have a long and polemical history. Often macroeconomists want to arrive at a set of business cycle facts that dynamic general equilibrium models of stationary economic fluctuations can aim to replicate [Canova, 1998]. In this framework, the trend component is supposed to represent the behavior of a data series absent the effects of business cycle fluctuations and, by implication, is inherently a counterfactual series.

We present an approach to business cycle filtering that uses a nonstructural vector autoregression (VAR) and estimates a counterfactual history of the multivariate system absent business cycle fluctuations. To do this, we use Markov Chain Monte Carlo methods to augment a standard macroeconomic VAR with a latent business cycle index that is negative during NBER recessions and positive during expansions. This data augmentation is the result of sampling values of a latent variable that governs a probit equation, using techniques pioneered in Albert and Chib (1993) and extended to a dynamic probit in Dueker (1999). The use of a vector autoregresssion to model how a limited dependent variable is determined stems from the Qual VAR of Dueker (2005). Given draws of the VAR coefficients and the latent business cycle index, we sample counterfactual values of the macroeconomic variables in the case where the latent busines cycle index is held constant. These counterfactual values represent posterior beliefs about how the economy would have evolved absent business cycle fluctuations. To be consistent with other filtering methods, one can refer to the counterfactual value as the "trend" level, without implying that interest rates, for example, contain a unit root. One advantage of our use of a VAR framework is that it provides model-consistent counterfactual values in the same way that VARs provide model-consistent forecasts, so data series are not detrended in isolation from each other. The fact that the business cycle index is an appended variable to a standard macroeconomic VAR system means that we do not have to identify a structural business cycle shock from a set of macroeconomic variables in order to 'shut down' business cycle fluctuations; instead, we calculate values of the macroeconomic data that would be consistent with a history in which the appended business cycle index remained constant.

Using a VAR to perform counterfactual analysis is not new. Leeper and Zha (2005) employ a structural VAR to investigate how the economy would have evolved if monetary policy shocks had been different at key junctures. Since Leeper and Zha introduce counterfactual monetary policy shocks to the federal funds rate, they need to simulate future systematic changes in the federal funds rate that would have taken place in this counterfactual case. For this reason, they need a structural VAR that identifies an impulse response function to monetary policy shocks. In our counterfactual simulation, in contrast, the future changes to the business cycle index are not in doubt. It is specified that the necessary mongrel VAR shocks take place to hold the business cycle index fixed across time. Thus, we have no need to separate these mongrel VAR shocks into structural components nor to identify an impulse response function.

The following section provides an overview of our business cycle filter and describes the Qual VAR model that forms the basis for the counterfactual analysis. The third section presents the trend estimates that remove the effects of business cycle fluctuations.

2 Outline of filtering procedure

Based on the view that business cycle filtering is inherently a counterfactual exercise, our procedure stems from a Qual VAR augmented with a business cycle index from Dueker (2005a). This latent business cycle index crosses zero by construction at NBER turning points. The distance of the business cycle index from zero indicates either the strength of an economic expansion or the depth of a recession. The counterfactual draws required for the filter are based on the counterfactual case where the business cycle index is held constant.

The NBER recession/expansion classifications are summarized by a 0/1 variable, denoted $NBER$. The latent business cycle index, z, is a continuous variable that lies behind the binary classifications:

$$\begin{aligned} NBER_t &= 0 \text{ if } z_t \leq 0 \text{ (recession)} \\ NBER_t &= 1 \text{ if } z_t > 0 \text{ (expansion)} \end{aligned}$$

Thus, we use the NBER recession/expansion classifications as data that imply that the latent business cycle index has a truncated (normal) distribution. This index will be set at a constant level in the counterfactual simulations in order to 'shut down' business cycle fluctuations.[1]

2.1 State-space form of Qual VAR

Let Y_t be a $k \times 1$ vector comprised of $(X_t', z)'$, where z is the scalar latent business cycle index and the $k-1$ elements of X are observed macroeconomic data series. The vector

[1]Kose, Otrok and Whiteman (2003) estimate world and regional business cycle indices in a dynamic factor model. Their model has autoregressive errors, as opposed to the autoregressive variables from the Qual VAR used here.

autoregressive process for Y is of order p:

$$Y_t = c + \sum_{i=1}^{p} \Phi^{(i)} Y_{t-i} + \epsilon_t, \tag{1}$$

where $\epsilon = (\epsilon_X', \epsilon_z)'$, and

$$\Phi^{(i)} = \begin{pmatrix} \Phi_{XX}^{(i)} & \Phi_{Xz}^{(i)} \\ \Phi_{zX}^{(i)} & \Phi_{zz}^{(i)} \end{pmatrix}.$$

As in other probit-type models, the variance of ϵ_z is normalized to one in the VAR covariance matrix denoted $\Sigma = \mathrm{Cov}(\epsilon)$. That is, the last row and column of Σ are parameterized as correlations and not covariances.

A useful state-space representation of the Qual VAR from eq. (1) has the following state equation, where lags of X, which will have zero Kalman gains, are not included in the state vector to facilitate smoothing:

$$\begin{aligned} \begin{pmatrix} X_t \\ z_t \\ z_{t-1} \\ \vdots \\ z_{t-p+1} \end{pmatrix} &= \begin{pmatrix} c_X \\ c_z \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \Phi_{XX}^{(1)} & \Phi_{Xz}^{(1)} & \Phi_{Xz}^{(2)} & \cdots & \Phi_{Xz}^{(p)} \\ \Phi_{zX}^{(1)} & \Phi_{zz}^{(1)} & \Phi_{zz}^{(2)} & \cdots & \Phi_{zz}^{(p)} \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X_{t-1} \\ z_{t-1} \\ z_{t-2} \\ \vdots \\ z_{t-p} \end{pmatrix} \\ &+ \begin{pmatrix} \Phi_{XX}^{(2)} & \cdots & \Phi_{XX}^{(p)} \\ \Phi_{zX}^{(2)} & \cdots & \Phi_{zX}^{(p)} \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} X_{t-2} \\ X_{t-3} \\ X_{t-4} \\ \vdots \\ X_{t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{X,t} \\ \epsilon_{z,t} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \end{aligned} \tag{2}$$

The measurement equation for the Qual VAR is simply

$$X_t = \begin{pmatrix} I_{k-1} & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} X_t \\ z_t \\ z_{t-1} \\ \vdots \\ z_{t-p+1} \end{pmatrix}. \tag{3}$$

This model is used in Kalman filtering and smoothing recursions to generate multi-move sampling of the latent variable z. The novelty of the approach is that the Kalman filter recursions are modified to take account of a state variable (z) that is truncated normal. The appendix discusses the procedure from Dueker (2005b) for Kalman filtering a truncated normal state variable. Multi-move sampling is highly efficient, relative to single-move sampling, in terms of convergence to the joint posterior distribution, because it induces much less serial correlation across draws of the latent variable z.

For the counterfactual draws, we essentially reverse the roles of X and z in the state equation. The counterfactual case that serves as the basis for the business cycle filter is one that treats z as the observable data and counterfactual X as the latent data to be inferred. The counterfactual values of X are denoted as $\widetilde{X}$ and the counterfactual shocks as $\widetilde{\epsilon}$.

$$\begin{pmatrix} \widetilde{X_t} \\ \overline{z} \\ \widetilde{X_{t-1}} \\ \widetilde{X_{t-2}} \\ \vdots \\ \widetilde{X_{t-p}} \end{pmatrix} = \begin{pmatrix} c_X \\ c_z \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \Phi_{XX}^{(1)} & \Phi_{Xz}^{(1)} & \Phi_{XX}^{(2)} & \cdots & \Phi_{XX}^{(p)} & 0 \\ \Phi_{zX}^{(1)} & \Phi_{zz}^{(1)} & \Phi_{zX}^{(2)} & \cdots & \Phi_{zX}^{(p)} & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & \cdots & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \widetilde{X_{t-1}} \\ \overline{z} \\ \widetilde{X_{t-2}} \\ \widetilde{X_{t-3}} \\ \vdots \\ \widetilde{X_{t-p-1}} \end{pmatrix}$$

$$+ \begin{pmatrix} \Phi_{Xz}^{(2)} & \cdots & \Phi_{Xz}^{(p)} \\ \Phi_{zz}^{(2)} & \cdots & \Phi_{zz}^{(p)} \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \overline{z} \\ \overline{z} \\ \overline{z} \\ \vdots \\ \overline{z} \end{pmatrix} + \begin{pmatrix} \widetilde{\epsilon_{X,t}} \\ \widetilde{\epsilon_{z,t}} \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4}$$

The counterfactual case relevant for the business cycle filter is the one where the latent business cycle index is held fixed throughout the sample period: $z_t = \overline{z}\ \forall t$. In practice, at a given MCMC iteration, $\overline{z}$ is taken to be the mean or median (or, as discussed below, some value proportional thereof) of the current draw of the latent variable vector, $\{z_t\}, t =$

$1, ..., T$, from the Qual VAR model of eqs. (1) and (2). In this way, the exercise is to infer counterfactual values of the macroeconomic variables when business cycle fluctuations are explicitly shut down in the counterfactual simulation. For the case where we are only interested in modeling the realization of the world that would be most consistent with a flat business cycle index, the measurement equation that accompanies state equation (4) is

$$\overline{z} = \begin{pmatrix} 0_{k-1} & 1 & 0_{k-1} & \cdots & 0_{k-1} \end{pmatrix} \begin{pmatrix} \widetilde{X_t} \\ \overline{z} \\ \widetilde{X_{t-1}} \\ \vdots \\ \widetilde{X_{t-p}} \end{pmatrix} \tag{5}$$

With the measurement equation (5), the realization of $\widetilde{X}$ that is most consistent (in terms of posterior likelihood) with a constant business cycle index is one where $\widetilde{X}$ is either nearly constant or on a nearly constant balanced growth path, depending on whether the variable in question trends upward. A useful alternative specification introduces a trade-off between the likelihood of the latent business cycle index, z, remaining fixed and the size of the deviations between the counterfactual values and the actual data X. That is, in the counterfactual model, the measurement equation can treat the (fixed) latent variable as the only observed data, or it also can include the historical data. In the former case, the only objective is to find the counterfactual values of X that were most likely to accompany a constant business cycle index z, even if these values greatly suppress the historical movements in X. In the latter case, the counterfactual values try to mimic the historical data, subject to the condition that the business cycle index remain steady. We can bridge these two cases by introducing to the measurement equation a parameter α, which equals zero for the former case and is greater than zero in the latter case:

$$\begin{pmatrix} \alpha X_t \\ \overline{z} \end{pmatrix} = \begin{pmatrix} \alpha I_{k-1} & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \widetilde{X_t} \\ \overline{z} \\ \widetilde{X_{t-1}} \\ \vdots \\ \widetilde{X_{t-p}} \end{pmatrix} + \begin{pmatrix} \alpha \eta_t \\ 0 \end{pmatrix}, \tag{6}$$

where $\alpha << 1$, so not too much weight is placed on making the counterfactual data look like the actual data; instead, a hard and fast requirement is that the counterfactual data be consistent with a fixed level of the latent business cycle index. Nevertheless, as α increases from zero, the counterfactual data will bend to make an attempt to mimic the actual data, at the expense of a lower marginal likelihood of a flat business cycle index. The covariance matrix, denoted Ω, of $\eta = X - \widetilde{X}$ is sampled as an inverted Wishart in a similar fashion to Σ. The trade-off that α introduces between having a smooth trend and having the trend fit the actual data is similar to the smoothing parameter that appears in the Hodrick-Prescott (1997) filter. Further comparison with the Hodrick-Prescott filter follows in the next section.

We set $\overline{z} = \gamma z^{mean}$ at each MCMC iteration. The parameter γ is chosen such that the sample mean of counterfactual output equals the sample mean of actual output. At this value of $\overline{z}$, the counterfactual sample means of the other variables in the system will differ from the actual sample means. We give a distinct treatment to the mean of counterfactual output versus interest rates, for example, because economists widely agree that business cycles, as commonly understood, do not affect the mean level of output, but there is no reason to assert that the existence of business cycles has no effect of the mean level of interest rates. With the business cycle come risk premia and monetary policy responses that can affect the sample mean of interest rates.

2.2 Comparison with Hodrick-Prescott filter

The counterfactual and multivariate nature and mean effect of business cycles on interest rates and inflation from the counterfactual filter stand in contrast to other statistical filters, such as the Hodrick-Prescott (HP) filter. For the HP filter, a state-space representation that uses the same notation is

$$\begin{pmatrix} \widetilde{X_t} \\ g_{t+1} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \widetilde{X_{t-1}} \\ g_t \end{pmatrix} + \begin{pmatrix} 0 \\ v_{t+1} \end{pmatrix} \tag{7}$$

$$X_{t+1} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \widetilde{X_t} \\ g_{t+1} \end{pmatrix} + e_{t+1}, \tag{8}$$

where $g_t = \Delta\widetilde{X_t}$ and the smoothing parameter $\lambda = \sigma_e^2/\sigma_v^2$.

A comparison in state-space form of the counterfactual business cycle filter with the HP filter makes some features readily apparent. First, it is clear that the HP filter will make the sample mean of the trend $\widetilde{X}$ equal to the sample mean of the data X for any variable. Furthermore, the likelihood function for HP is trying to fit white-noise measurement errors e in eq. (8), so the HP filter will trade off trend innovations v to try to keep the sub-sample means of $X - \widetilde{X}$ close to zero. The HP filter is also inherently univariate because this definition of the smoothing parameter λ requires scalar inputs.[2] Consequently, the trade-off implicit in the HP filter refers only to the variable in question: balancing $X - \widetilde{X}$ deviations against $\Delta\widetilde{X}$ (trend) innovations. One feature of this trade-off for the HP filter is that the penalty for bends in the trend does not depend on the timing of the bends. Thus, the two-sided HP filter will tend to make these trend changes lead the business cycle in order to prevent large gaps between the actual and trend values from opening. In addition, with the HP filter, one has to take it on faith that a propitious

[2]Kozicki (1999) discusses HP filtering of cointegrating combinations of variables as a quasi-multivariate extension.

choice of λ will lead to a measure of $X - \widetilde{X}$ that isolates the effect of business cycle fluctuations on X.

The counterfactual filter also penalizes $X - \widetilde{X}$ deviations when $\alpha > 0$, but the counterfactual filter has a less fungible trade-off: the business cycle index must remain fixed at $\bar{z}$ regardless. The trade-off allows counterfactual paths with a lower *likelihood* of the business cycle index remaining fixed, rather than allow variation in the counterfactual business cycle index. Too much fluctuation in the counterfactual macroeconomic data series would drastically lower the marginal likelihood of having a fixed business cycle index. In addition, the timing of movements in the counterfactual path cannot adjust to keep the gap between the actual and counterfactual values small without harming the marginal likelihood of a fixed business cycle index. Some movement in the counterfactual path is possible, however, especially in our multivariate framework. For example, suppose that output is well below trend. In the counterfactual model, the growth rate of counterfactual output could increase for a time and still be consistent with a fixed business cycle index if, for instance, the counterfactual interest rate were to move in a countervailing direction.

2.3 MCMC Sampling Scheme

The parameters were divided into four blocks for MCMC sampling via Gibbs: (i) VAR coefficients, denoted Φ; (ii) and (iii) covariance matrices, Σ and Ω; (iv) the latent business cycle index z. Markov Chain Monte Carlo estimation of this model consists of a sequence of draws from the following conditional distributions, π, where superscripts indicate the iteration number:

$$\begin{aligned} \text{VAR coefficients} \quad &\sim \quad \text{Normal} \\ \pi(\Phi^{(i+1)} \quad &| \quad \{z_t^{(i)}\}_{t=1,..,T}, \{X_t\}_{t=1,..,T}, \Sigma^{(i)}) \end{aligned}$$

$$
\begin{aligned}
\text{latent variable} &\sim \text{truncated Normal} \\
\pi(\{z_t^{(i+1)}\}_{t=1,..,T} &\mid \{X_t\}_{t=1,..,T}, \{NBER_t\}_{t=1,..,T}\Phi^{(i+1)}, \Sigma^{(i)}) \\
\text{Covariance matrix of VAR residuals} &\sim \text{inverted Wishart} \\
\pi(\Sigma^{(i+1)} &\mid \{z_t^{(i+1)}\}_{t=1,..,T}, \{X_t\}_{t=1,..,T}, \Phi^{(i+1)}) \\
\text{Counterfactual draws of maro variables} &\sim \text{Normal} \\
\pi(\{\widetilde{X}^{(i+1)}\} &\mid \overline{z}^{(i+1)}, \{X_t\}_{t=1,..,T}, \alpha, \Omega^{(i)}) \\
\text{Covariance matrix of counterfactual deviations} &\sim \text{inverted Wishart} \\
\pi(\Omega^{(i+1)} &\mid \{\widetilde{X}_t^{(i+1)}\}_{t=1,..,T}).
\end{aligned} \tag{9}
$$

To sample the VAR coefficients, we used a prior from Robertson and Tallman (2001) which puts considerable prior weight on the VAR system having an autoregressive root near unity.[3] Sims and Zha (1998) introduced this updated version of the Litterman (1986) unit-root prior. The covariance matrices are sampled from the inverted Wishart distribution, as discussed in Chib and Greenberg (1996, p. 428):

$$
\Sigma^{-1} \mid \Phi, Y \sim \mathcal{W}_k(T, \sum_t \epsilon_t \epsilon_t'). \tag{10}
$$

As in all probit-type models, the variance of the latent variable z is not identified and must be normalized to an arbitrary value. Consequently, elements of the last row and column of Σ are normalized to preserve correlations when the lower right-hand element is set to unity. In contrast, the covariance matrix Ω of $X - \widetilde{X}$ does not have any restrictions. A multi-move sampling algorithm for the latent variable is discussed in the appendix.

[3] We thank Ellis Tallman for providing code to draw these VAR coefficients.

3 Estimates of business cycle filtered data from the counterfactual approach

3.1 Application to a small macro system

The VAR includes five monthly data series from January 1959 to March 2005: the industrial production index (in logs); consumer price inflation; the term spread between the 10-year Treasury yield and the 3-month Treasury bill rate; the monthly average of the federal funds rate *plus* the latent business cycle index. The lag length was not the subject of any model selection procedure; instead, with monthly data, we chose 12 lags to ensure that seasonality was not affecting our measurement of the business cycle. Although we used a prior that put weight on having a large autoregressive root in the VAR, we rejected randomized draws of VAR coefficients at each MCMC iteration where the largest autoregressive root exceeded unity to rule out explosive roots. The value of γ that set the posterior sample mean of counterfactual output (industrial production) essentially equal to the sample mean of actual output was 0.95.

The counterfactual history we derive represents high posterior density estimates of how the economy would have behaved absent any business cycle fluctuations, i.e., with the business cycle index variable held fixed. To show the fluctuations that we are shutting down in the counterfactual analysis, Figure 1 plots the posterior mean of the business cycle index, z, with NBER recession periods shaded. The case where the business cycle index is constant is considered a world free of business cycle fluctuations for the purposes of this explicitly counterfactual detrending procedure. The business cycle index crosses zero at NBER turning points by construction. The business cycle index correctly identifies which recessions were the most severe: 1974-75, 1981-82 and 1980. (The latter was short but deep.)

Figure 2 graphs industrial production and the posterior mean of two counterfactual series: one with α set to zero and one with α set to 0.01.[4] Figure 2 highlights the way in which setting α above zero causes the counterfactual series to follow the actual data more closely, although at the expense of lowering the marginal likelihood of observing a flat business cycle index. Figure 3 compares counterfactual industrial production given $\alpha = 0.01$ with the HP trend given $\lambda = 128,800$, a value recommended for monthly data. The bends in the HP trend clearly keep the trend line closer to the actual data than does the counterfactual filter. For comparisons in the paper with other filters, such as HP, we concentrate on results for the counterfactual filter with $\alpha = 0.01$ and not $\alpha = 0$ because, although a world with almost no fluctuations whatsoever is most consistent with a world with no business cycle fluctuations, we do not wish to attribute nearly all fluctuations to the business cycle.

Figure 4 shows the growth rates in the counterfactual industrial production paths. With $\alpha = 0$, the counterfactual growth path is nearly linear, apart from a gradual decline in the trend growth rate that gradually achieves a trend growth slowdown by the early 1970s and roughly coincides with the well-known productivity slowdown. Perron and Wada (2005) argue that a single structural break in linear trend growth occurred in 1973 and that other filtering methods, such as Beveridge and Nelson (1981), are overparameterized representations of post-war U.S. data. With $\alpha = 0.01$, rather than zero, the counterfactual growth rate undergoes a much greater range of fluctuation and often exceeds the fluctuations in HP trend growth.[5] The correlation between the two measures of trend growth is 0.64. Nevertheless, a key difference in Figure 3 is that the fluctuations in HP trend growth seem to precede similar fluctuations in counterfactual trend. Both filters

[4] A simple rule of thumb we used in choosing α is that if α is set too high, a negative counterfactual value of the interest rate can occur.

[5] Note that, apart from some small differences at the ends of the sample, the first differences in the HP trend are identical to the HP trend of the first differences of the data due to the linear nature of the filter.

are two-sided and both incorporate an objective to keep the trend close to the actual data, but the HP trend conforms with this objective to a much greater extent than the counterfactual trend. Figure 5 depicts the running means of the output gaps—the difference between industrial production and the filter-implied trend level—for the counterfactual and HP filters. Within the first 10 percent of the sample, the HP filter starts to keep the running mean output gap within a narrow band near zero, whereas the counterfactual filter does not. As a two-sided filter, HP takes into account a boom or recession in the subsequent data and adjusts the trend in order to minimize the extent to which the boom or recession will cause actual output to depart from trend. In other words, HP penalizes bends in trend but allows the timing of any trend shifts to be geared towards minimizing the deviations between the data and the trend. The counterfactual filter, in contrast, penalizes bends in trend that make it less likely that no business cycle fluctuations took place in the counterfactual case. The timing of shifts in the counterfactual trend responds more to the marginal likelihood of no business cycle fluctuations in z than to keeping the running mean of counterfactual output close to actual output.

Figure 6 plots the period-to-period changes in the trend growth rates from the counterfactual and HP filters. The trend growth rate is subject to much more abrupt changes from the counterfactual filter than the HP filter. Figure 7 plots the business cycle index with the period-to-period changes in the HP trend growth rate lagged 24 months. At this lag length, the correlation between the two is at its highest, 0.60, which suggests that the two-sided HP filter starts raising its trend growth rate two years in advance of a boom in order to prevent the boom from causing a large gap between actual and trend output. Thus, the HP filter adjusts the timing of trend shifts in order to optimize its trade-off between penalizing changes in trend and deviations of the series from its HP trend. For the counterfactual filter, in contrast, the contemporary correlation is the highest one (0.44) between the business cycle index and changes in the trend growth rate of output. The counterfactual filter accords with the view that recessionary periods are a confluence of permanent and transitory shocks. Although the counterfactual filter also penalizes deviations between the series and its counterfactual level, it cannot adjust the timing of trend shifts to minimize these deviations without lowering the marginal likelihood of a fixed business cycle index.

Many studies summarize the cyclical behavior of an economy through a table of correlations of business cycle filtered data. Table 1 presents such a cyclical correlation table that compares the counterfactual filter, the HP filter and the band pass (BP) filter that is discussed in section 3.2 below. Often such tables have no consistent "numeraire" variable across models and/or filtering methods. In our case, however, we are able to compare, for each filtering method, the correlation between the filtered data and a variable that does not depend on the filter— the business cycle index estimated from the Qual VAR. Cogley and Nason (1995) warn about apples-to-oranges comparisons between filters that take place when there is no numeraire series to the comparison that does not depend on the filter. In comparison with the HP and BP filters, the counterfactual filter indicates that inflation is acyclical, the term spread is highly procyclical, and the federal funds rate is less countercyclical. It is also interesting to note that trend output growth is less procyclical from the counterfactual filter than from the HP filter. This correlation confirms the observation from Figures 3 and 4 that trend growth from the HP filter undergoes sizable swings that coincide with the business cycle, even though the effects of the business cycle are supposed to be filtered out of the trend measure.

Figures 8 and 9 show the counterfactual values (posterior means) of CPI inflation, the federal funds rate and the term spread, with α set both to zero and to 0.01, vis-a-vis the actual data. The graphs also include the HP trends with $\lambda = 128,800$. Trend inflation and interest rates from the counterfactual filter never reach the levels that their HP trends do. Figure 10 plots the implied trend real rate of interest on federal funds from both the counterfactual and HP filters. The implied HP trend real rate experiences sharper swings and goes below zero in the late 1970s and early 2000s. The counterfactual real rate, in contrast, never goes below zero, although it is at an unprecedentedly low level by 2005. Similary, in Figure 11, the counterfactual term spread between 10-year Treasury bonds and the 3-month Treasury bill never dips close to zero, which suggests that only the effects of the business cycle lead to a flat or inverted yield curve. The HP trend for the term spread, in contrast, dips close to zero following several recessions and has even inverted once.

3.2 Counterfactual filtering of random walk series

A desirable property of a business cycle filter is that it would not find spurious cycles in a random walk series. Suppose that we included in the multivariate counterfactual filter a random walk series that is only contemporaneously correlated with the business cycle index, with no other lag/lead relationship. Cogley and Nason (1995) and Murray (2003) show that the univariate HP and band pass filters, respectively, are not designed to return a white-noise series when a random walk is filtered. Hence, the HP and BP filters find spurious cycles when presented with random walk inputs. With a large value of λ, the HP filter, for example, must imply a smooth trend, even for a series that does not have a smooth trend. We suggest that, while the counterfactual filtering approach will not mechanically return white noise from such a random walk series, judicious use of diagnostics from the Qual VAR that lies behind the approach can lead to the proper inference about a random walk input.

If the random walk series is only contemporaneously correlated with the business cycle index, the true VAR coefficients on its lags in the business cycle index equation are all zero. In practice, however, these VAR coefficients are not exactly zero across MCMC iterations, so that a smooth counterfactual trend will be more consistent with a constant business cycle index. Hence, mechanically the filter will return a posterior mean for the counterfactual trend that is spuriously smooth, like HP and BP. However, if one looks at the distribution of the VAR coefficients on lags of the random walk series in the business cycle index equation, their distribution will be centered on zero. Thus, one can restrict them to zero and the implied counterfactual trend, given $\alpha > 0$, will differ, by construction, from the actual data by just a white-noise shock—exactly the way one would want to detrend a random walk. We added a random walk series to the Qual VAR model behind the counterfactual filter and found that the VAR coefficients on the random walk in the business cycle index equation were, in fact, centered on zero. Thus, the counterfactual approach, with its econometric model of a business cycle index, helps identify which variables are not really related to the business cycle, such as the generated random walk, in which case the counterfactual series ought to differ from the actual data by white noise only.

3.3 Business cycle filter viewed in frequency domain

A widely-held view is that business cycle fluctuations consist of cycles that last between 18 months and 8 years. In this vein, Baxter and King (1999) aim to measure business cycle components in macroeconomic data series with a band pass filter that ideally has a gain of unity in the frequency range between 18 months and 8 years and zero elsewhere, where gain reflects the factor by which the spectrum at a given frequency is raised or lowered by the filter.[6] By comparison, at recommended values of the smoothing parameter λ, the HP has a gain near zero for frequencies lower than 8 years per cycle and the gain rises

[6] Orphanides and van Norden (2002) and Christiano and Fitzgerald (2003) modify the BP filter to focus on the behavior near the end of the sample.

quickly to reach unity by the 18 months-per-cycle frequency.

Here we check whether the variance in the estimated business cycle index, z, is due to cycles in this frequency range by plotting the spectrum of the posterior mean of the business cycle index in Figure 12, where the plotted periodograms are a three-ordinate moving average with triangular weights. Figure 9 also include shading to indicate the 18 month to 8 year frequency range, and it is noteworthy that most of the spectral power of the business cycle index lies in the shaded business cycle range. This finding does not necessarily corroborate the use of the BP filter as a business cycle filter on data, however. The fact that the spectral power of the business cycle index z is concentrated within that spectral range dose not imply that the *effect* of business cycle fluctuations on macroeconomic data series is limited to that spectral range because there could be lag/lead relationships between the business cycle index and its impact on observed data series. In fact, the VAR coefficients on lagged values of the business cycle index are capable of creating a wide variety of phase shifts in the effect of business cycle fluctuations on the observed macroeconomic data series.

Therefore Figure 12 also includes plotted spectra of the counterfactual business cycle components of the four data series: industrial production, inflation, the federal funds rate and the term spread. For the term spread—a well-known business cycle indicator—the periodogram of the business cycle component matches that of the business cycle index quite closely, with almost no phase shift. The same is roughly true for the federal funds rate as well. For output and inflation, however, a nontrivial portion of the spectral power of the business cycle components occurs at frequencies lower than 8 years per cycle. The coefficients on lagged values of the business cycle index in the Qual VAR create idiosyncratic transfer functions that do not correspond to a one-size-fits-all univariate filter. In sum, this frequency domain view bolsters the idea that one-size-fits-all univariate statistical filters are likely to be inadequate at isolating the effect of the business cycle on macroeconomic data series.

We note that other approaches to business cycle detrending allow business cycles to have idiosyncratic effects across different macroeconomic variables. Blanchard and Quah (1989) identify demand and supply shocks in a bivariate system under the assuption that demand shocks have a temporary effect on the level of output, and thus do not conribute to trend movements. Cochrane (1994) relies on consumption changes as a measure of innovations to trend output. The Beveridge-Nelson (BN, 1981) decomposition defines the trend level today as the long-run forecast of the level of a series (minus its deterministic trend). The BN decomposition thus provides a trend measure that conditions on past but not future shocks. Proietti and Harvey (2000) extended the Beveridge-Nelson approach to a two-sided trend estimate with Kalman smoothing. In its favor, the BN approach can be applied to a multivariate system, so trend esimtates are consistent across series, as in King, Plosser, Stock and Watson (1991) and Ariño and Newbold (1998). Univariate trend-cycle decompositions from the Kalman filter appear in Clark (1987) and Harvey and Jaeger (1993). Morley, Nelson and Zivot (2003) demonstrate that the Beveridge-Nelson decomposition is equivalent to a state-space decomposition where the innovations to trend and cycle are correlated. The upshot is that, rather than viewing statistically filtered data as a data source for estimated macroeconomic models, greater emphasis should be placed on using empirical macroeconomic models to identify the business cycle components in the data. Currently much of the emphasis in macroeconomic modeling is on identifying structural shocks and impulse responses, given measures of business cycle components from mechanical filters, whereas it might be fruitful to use model-consistent Qual VAR coefficients to measure business cycle components of the data.

4 Conclusions

This article introduces a method of business cycle filtering by way of counterfactual analysis of a vector autoregression augmented with a latent business cycle index, the sign of which corresponds to NBER business cycle classifications. We use a novel multi-move Bayesian sampling scheme to account for the truncated nature of the business cycle index. By holding the business cycle index fixed at each iteration of the Markov Chain Monte Carlo simulation, we perform a counterfactual analysis of how the economy would have behaved absent business cycle fluctuations. The counterfactual simulation within a VAR framework provides model-consistent counterfactual paths across series. This multivariate framework helps avoid the implication from the Hodrick-Prescott filter that the business cycle filtered real interest rate dipped below zero on occasion.

A key practical difference between the counterfactual businesss cycle filter and the Hodrick-Prescott filter is that the latter forces the running mean of the cyclical component of the data to remain near zero. To achieve this, the HP trend must bend in advance of long-lasting booms and recessions. The counterfactual filter, in contrast, does not force the running means of the actual and counterfactual data to remain so close to each other. The emphasis with the counterfactual filter is to find counterfactual paths for the data that are consistent with a flat business cycle index, even if these paths imply relatively persistent departures from the actual data.

Analysis of the relation between the business cycle components (filtered data) in the frequency domain suggests that the counterfactual detrending approach is not consistent with one-size-fits-all mechanical filters. Although the spectral power of the business cycle index is concentrated within the spectral range associated with the band pass filter, we find that the *effect* of business cycle fluctuations on macroeconomic data series is not limited to that spectral range because of phase shifts between the business cycle fluctuations and their impacts on observed macroeconomic series. In particular, the phase shift imparts greater low-frequency variance to macro variables. Thus, the counterfactual detrending approach provides complementary information regarding empirical business cycle facts in the spirit of Canova (1998), and we highlight this in lag/lead correlation tables of the type used in many studies where model-simulated data are matched to the empirical "business cycle facts."

This method of counterfactual analysis could also prove of value in examining other questions, such as a counterfactual simulation where the central bank never lifted a finger toward intervening in foreign exchange markets.

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Appendix: Multi-move sampling of latent business cycle index

Based on Dueker (2005b), consider a macroeconometric model written as a p-order vector autoregression, where z is the latent variable that lies behind the qualitative data, z^q, and the observed macroeconomic time seris, X:

$$Y_t = c_Y + \sum_{i=1}^{p} \Phi^{(i)} Y_{t-i} + \epsilon_t, \tag{11}$$

where $Y_t = (X_t', z_t)'$ is $k \times 1$ and

$$\Phi^{(i)} = \begin{pmatrix} \Phi^{(i)}_{XX} & \Phi^{(i)}_{Xz} \\ \Phi^{(i)}_{zX} & \Phi^{(i)}_{zz} \end{pmatrix}.$$

A standard state-space representation of the VAR has the following state equation:

$$\begin{pmatrix} Y_t \\ Y_{t-1} \\ \vdots \\ Y_{t-p+1} \end{pmatrix} = \begin{pmatrix} c_Y \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \Phi^{(1)} & \Phi^{(2)} & \cdots & \Phi^{(p)} \\ I & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ 0 & \cdots & I & 0 \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ Y_{t-2} \\ \vdots \\ Y_{t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{Y,t} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

The measurement equation for the Qual VAR is simply

$$X_t = \begin{pmatrix} I_{k-1} & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} Y_t \\ Y_{t-1} \\ \vdots \\ Y_{t-p+1} \end{pmatrix} \tag{12}$$

To achieve multi-move sampling, it is necessary to derive the conditional distribution of the entire vector of the latent variable conditional on the parameter vector, Θ, the observed qualitative data, z^q, and the other macroeconomic data, X_t:

$$\pi(\{z_i\}_{i=1}^T \mid \Theta, \{z_i^q\}_{i=1}^T, \{X_i\}_{i=1}^T).$$

We can summarize the above state-space model (suppressing the constants) as:

$$
\begin{aligned}
X_t &= HS_t \\
S_t &= FS_{t-1} + \epsilon_t
\end{aligned} \tag{13}
$$

The number of rows in the state vector is k and F_k denotes row k of F.

Without loss of generality, assume that z is the last element in the state vector S. Because we want to allow for the possiblity that the covariance matrix of ϵ_t has non-zero off-diagonal elements, let

$$\epsilon_t = W\eta_t,$$

where W is an upper-triangular Cholesky decomposition of the covariance matrix Q of ϵ_t. The truncation from having z_t^q in category j implies that z_t lies in the range (d_j, d_{j+1}), and this truncation of z will change the mean and variance of the last element of η from the unconditional values of zero and one, respectively.

The mean of ϵ, conditional on trunction, becomes

$$E[\epsilon_{t+1} \mid z_{t+1}^q] = W \begin{pmatrix} 0 \\ \vdots \\ 0 \\ a \end{pmatrix},$$

where we denote $\alpha_{j+1} = W^{-1}[z_{t+1} - F_k S_{t|t} + d_{j+1}]$, $\alpha_j = W^{-1}[z_{t+1} - F_k S_{t|t} + d_j]$ such that

$$a = \frac{\phi(\alpha_{j+1}) - \phi(\alpha_j)}{\Phi(\alpha_{j+1}) - \Phi(\alpha_j)}.$$

and the variance of ϵ, conditional on truncation, becomes

$$\tilde{Q} = W \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & \cdots & 1 & 0 \\ 0 & \cdots & 0 & v \end{pmatrix} W',$$

where

$$v = 1 - a^2 - \frac{\alpha_{j+1}\phi(\alpha_{j+1}) - \alpha_j\phi(\alpha_j)}{\Phi(\alpha_{j+1}) - \Phi(\alpha_j)}.$$

Kalman filter recursions

The Kalman filter with the truncated state variable proceeeds as follows:

1. Data forecast error (DFE) conditional on z^q:

$$y_{t+1} - y_{t+1|t} = y_{t+1} - HFS_{t|t} - HE[\epsilon_{t+1} \mid z^q_{t+1}]$$

2. State forecast variance:

$$P_{t+1|t} = FP_{t|t}F' + \tilde{Q}$$

3. Data forecast error variance:

$$\mathrm{Var}_t(DFE_{t+1}) = HP_{t+1|t}H'$$

The equations to update the state and the precision matrix are based on

$$S_{t+1|t+1} = S_{t+1|t} + E[(S_{t+1} - S_{t+1|t})DFE_{t+1}] \times [\mathrm{Var}_t(DFE_{t+1})]^{-1}DFE_{t+1}.$$

Nevertheless, because

$$E[(S_{t+1} - S_{t+1|t})DFE_{t+1}] = P_{t+1|t}H',$$

the Kalman filter update equations take the usual form, although the truncation information alters the data forecast error (DFE) and forecast variance inputs:

$$S_{t+1|t+1} = S_{t+1|t} + P_{t+1|t}H'[HP_{t+1|t}H']^{-1}DFE_{t+1} \quad (14)$$

$$P_{t+1|t+1} = P_{t+1|t} - P_{t+1|t}H'[HP_{t+1|t}H']^{-1}HP_{t+1|t} \quad (15)$$

Thus, these modifications to the Kalman filter recursions show that the Kalman filter remains a useful tool to calculate conditional densities in the case where one or more of the state variables is truncated normal. After the modified Kalman filtering, the usual (no modification necessary) Kalman smoothing equations can be applied in order to draw values of z backwards from the end of the sample, with the net result being a draw from the density $\pi(\{z\} \mid \{z^q\}, \{X\}, \text{parameters})$.

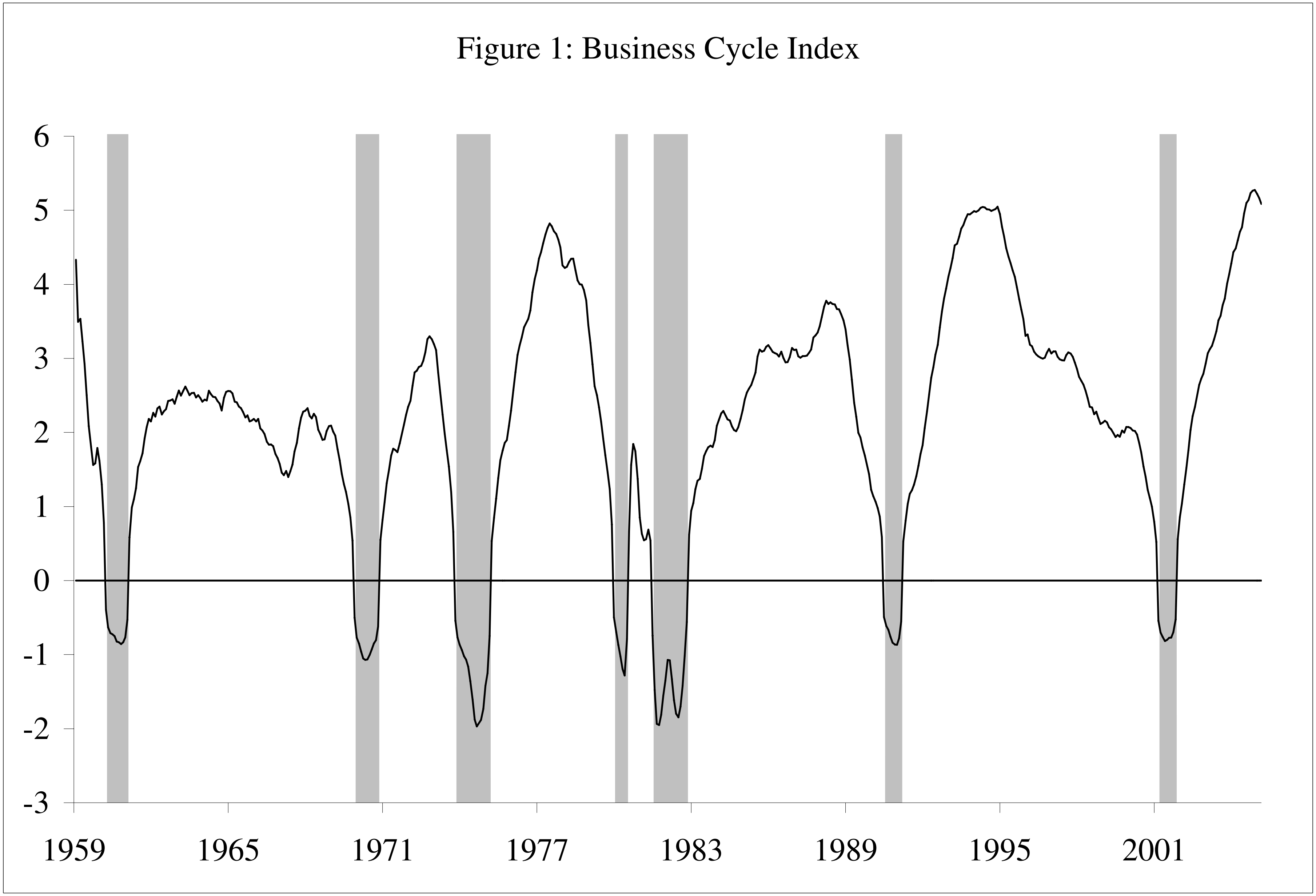


Figure 2: Industrial Production and Counterfactual Levels

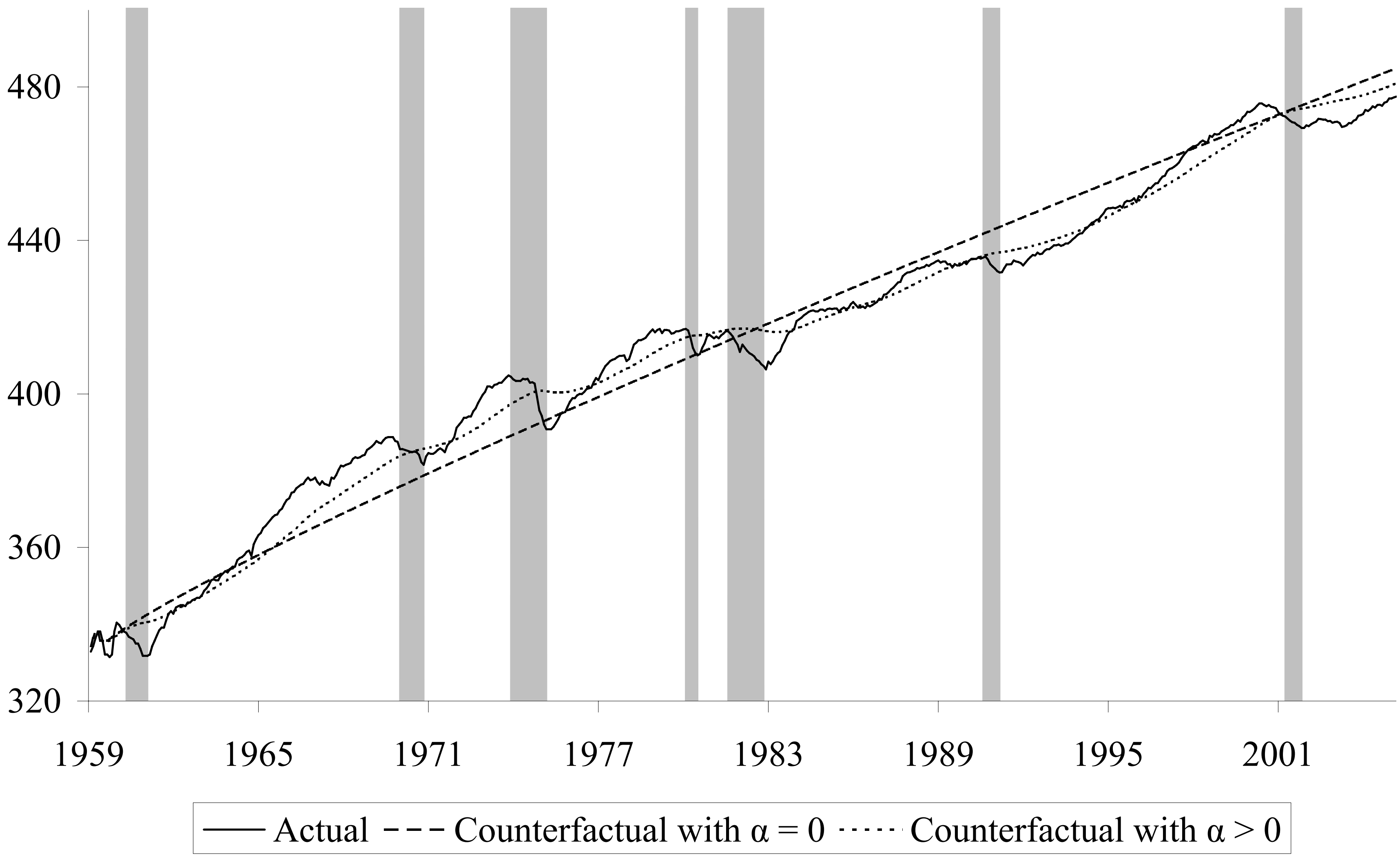
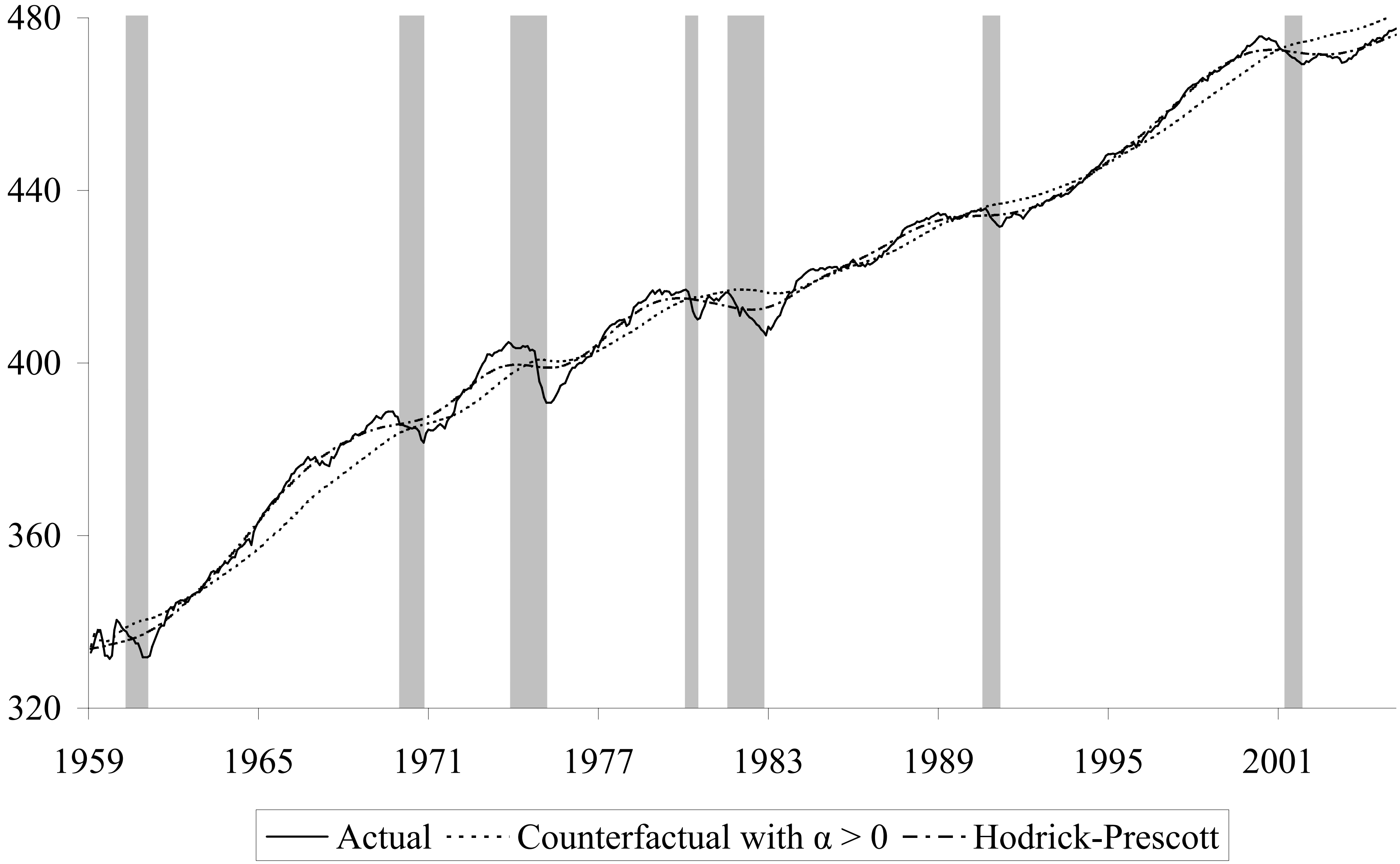


Figure 3: Counterfactual Industrial Production with Hodrick-Prescott Trend



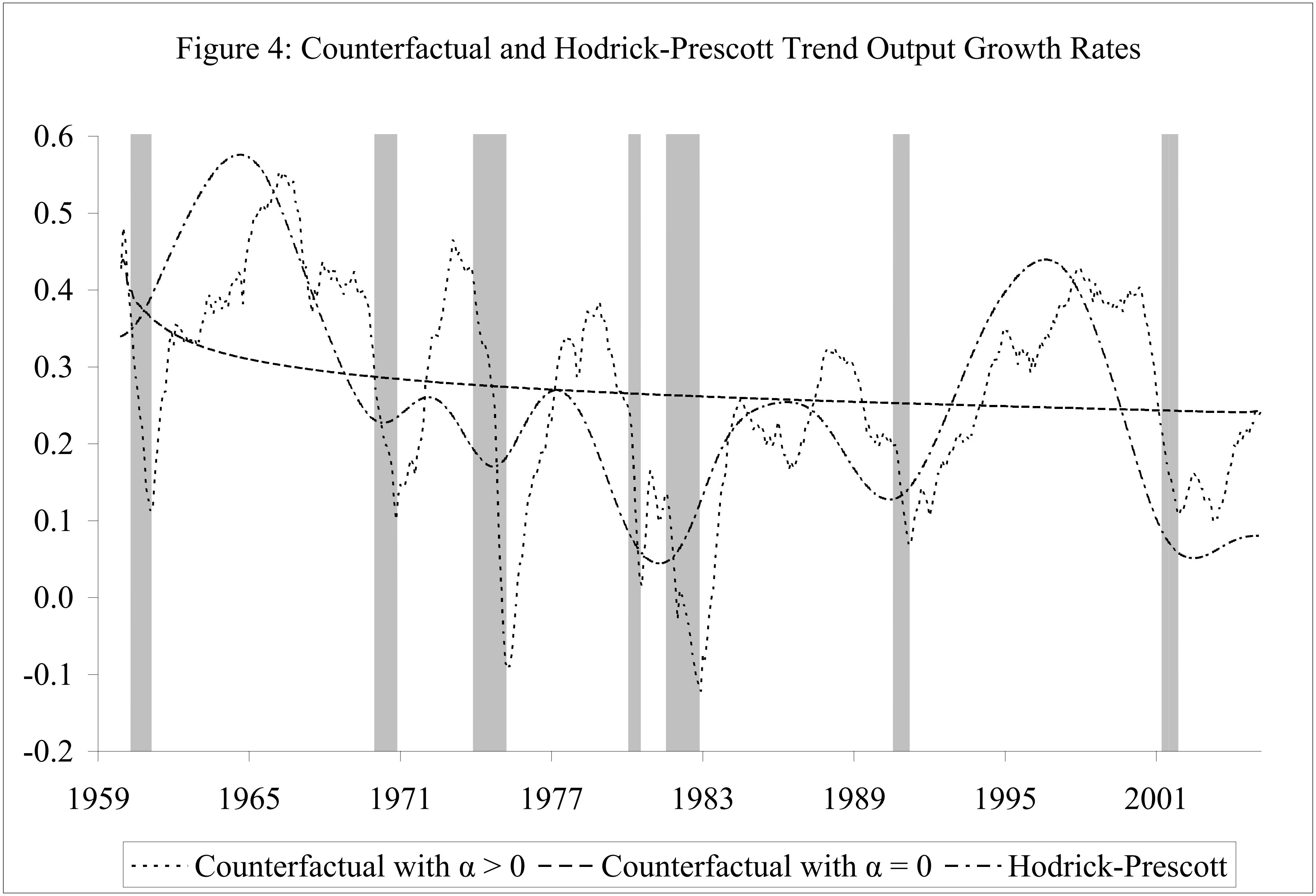
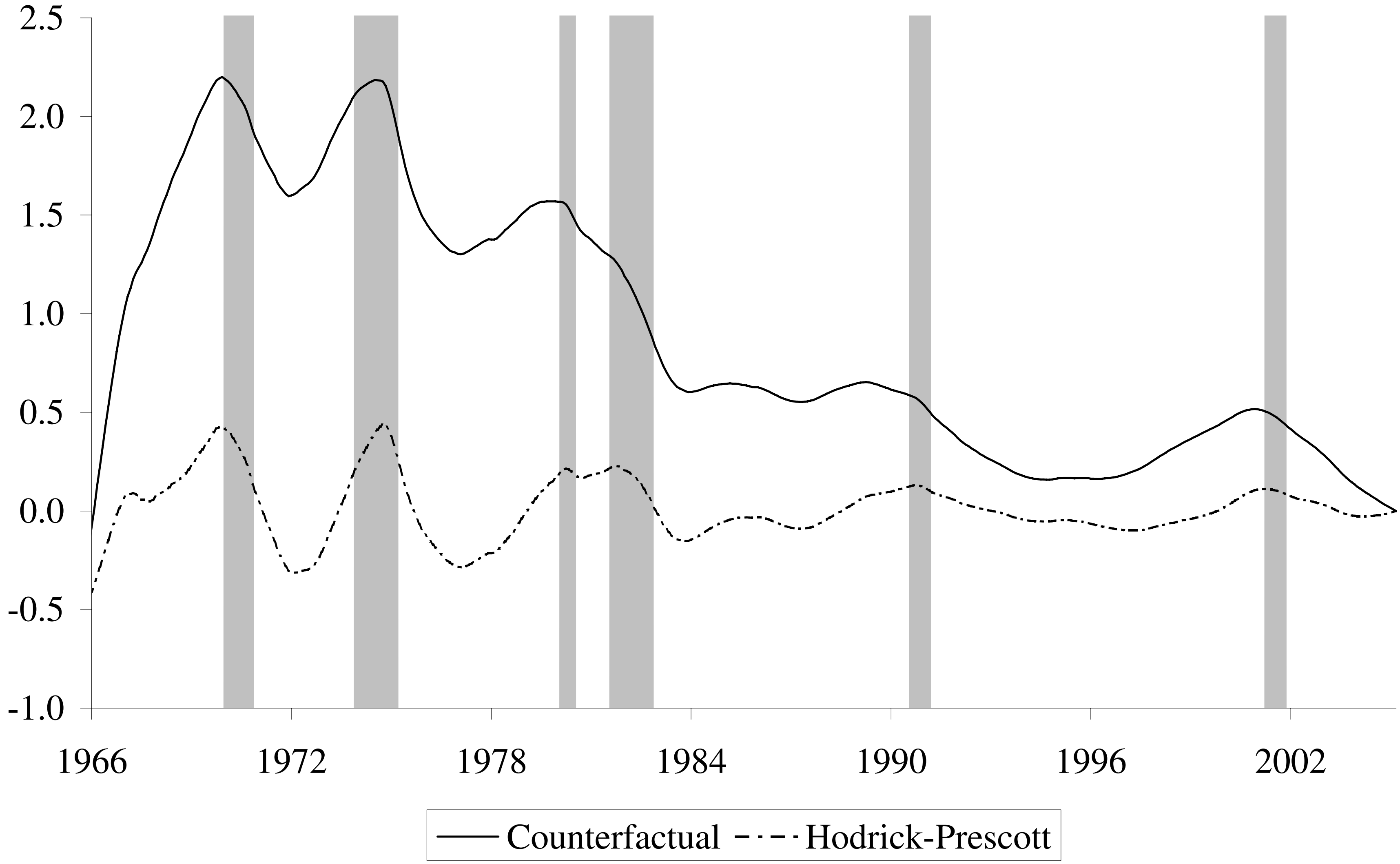


Figure 4: Counterfactual and Hodrick-Prescott Trend Output Growth Rates
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
1959
1965
1971
1977
1983
1989
1995
2001
Counterfactual with α > 0
Counterfactual with α = 0
Hodrick-Prescott

Figure 5: Running Means of Counterfactual and Hodrick-Prescott Output Gaps



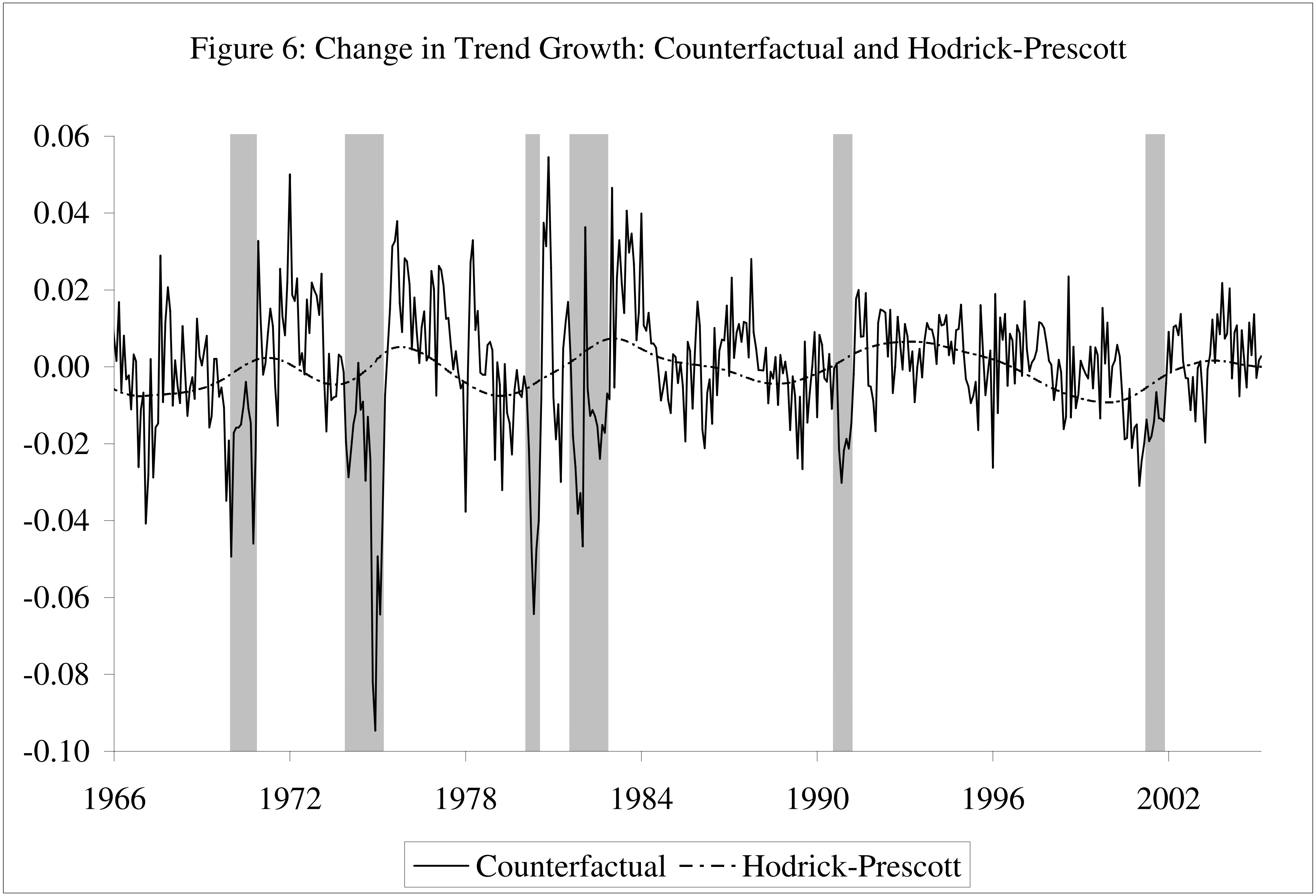


Figure 6: Change in Trend Growth: Counterfactual and Hodrick-Prescott
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
1966
1972
1978
1984
1990
1996
2002
Counterfactual
Hodrick-Prescott

Figure 7: Business Cycle Index and Changes in Hodrick-Precott Trend Growth Rate (Lagged 24 Months)

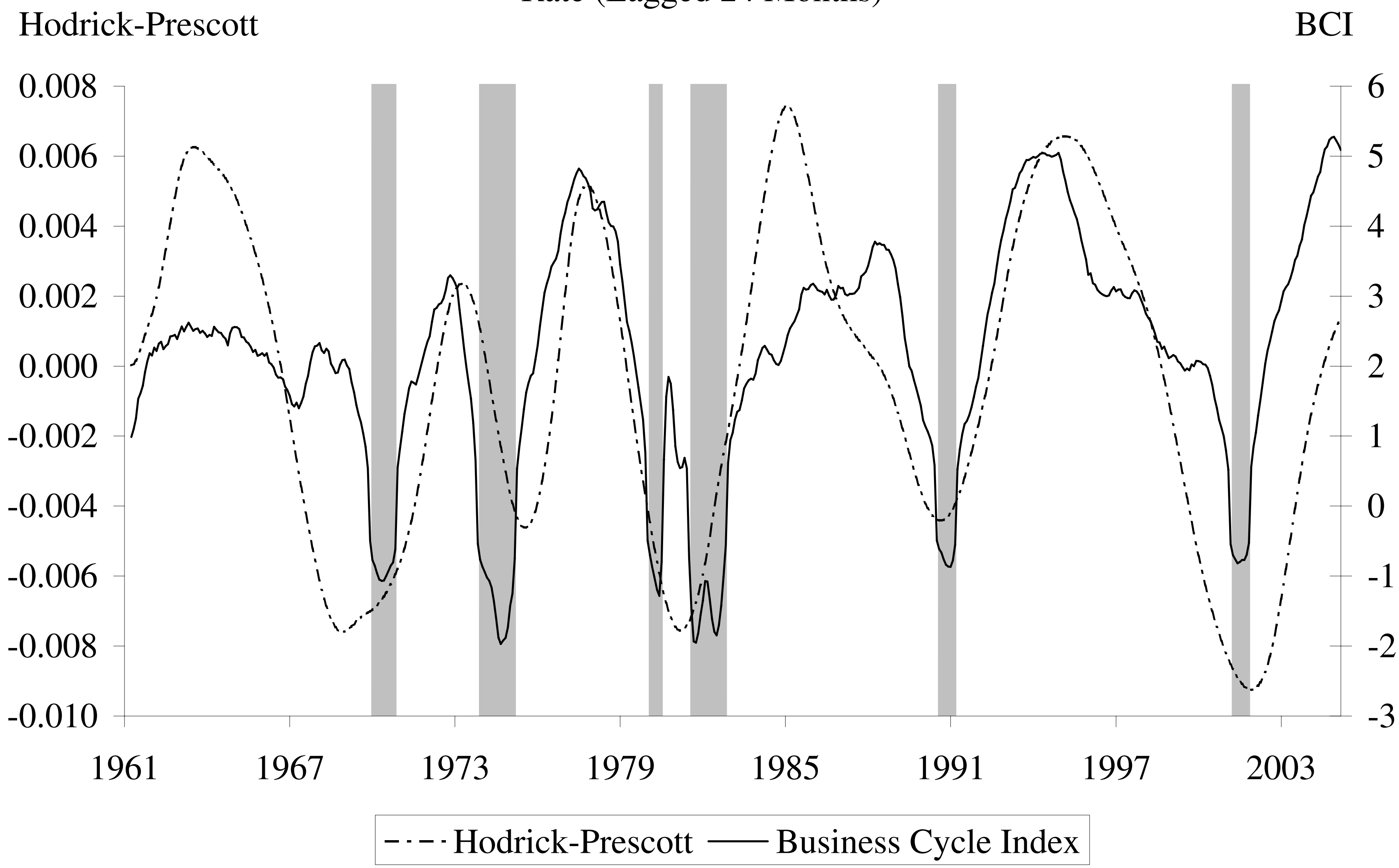


Figure 8: PCE Inflation (Monthly)

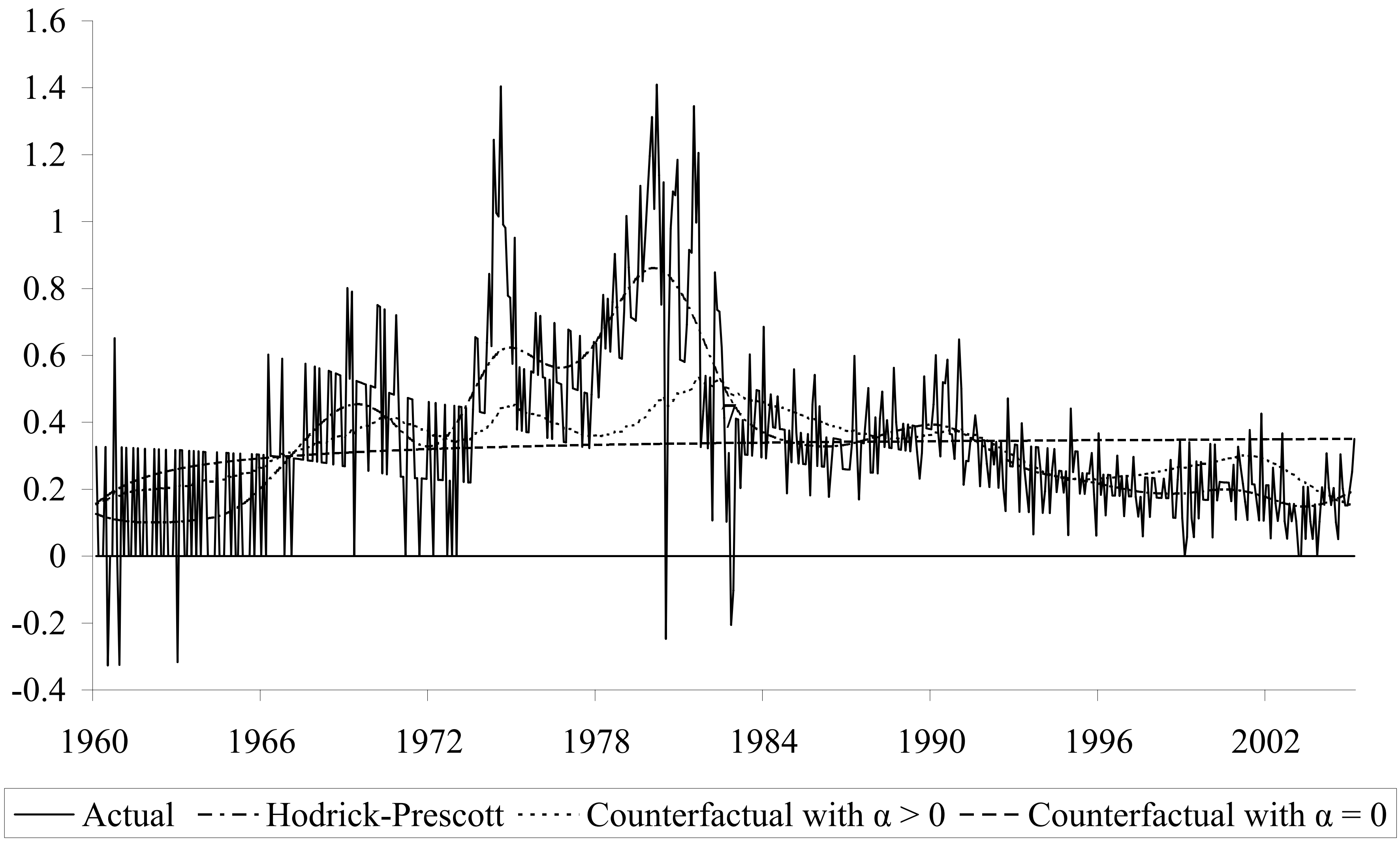


Figure 9: Federal Funds Rate

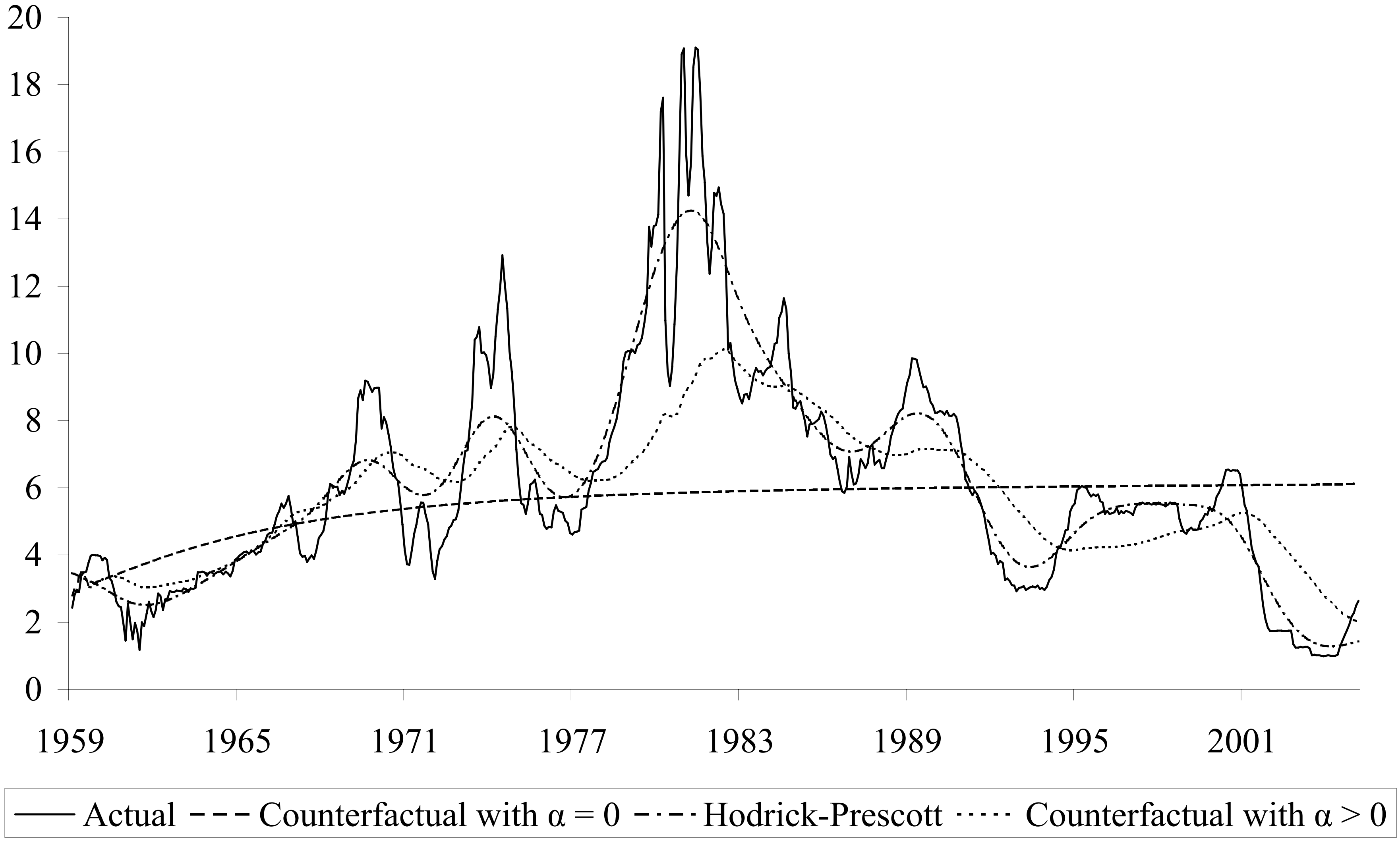
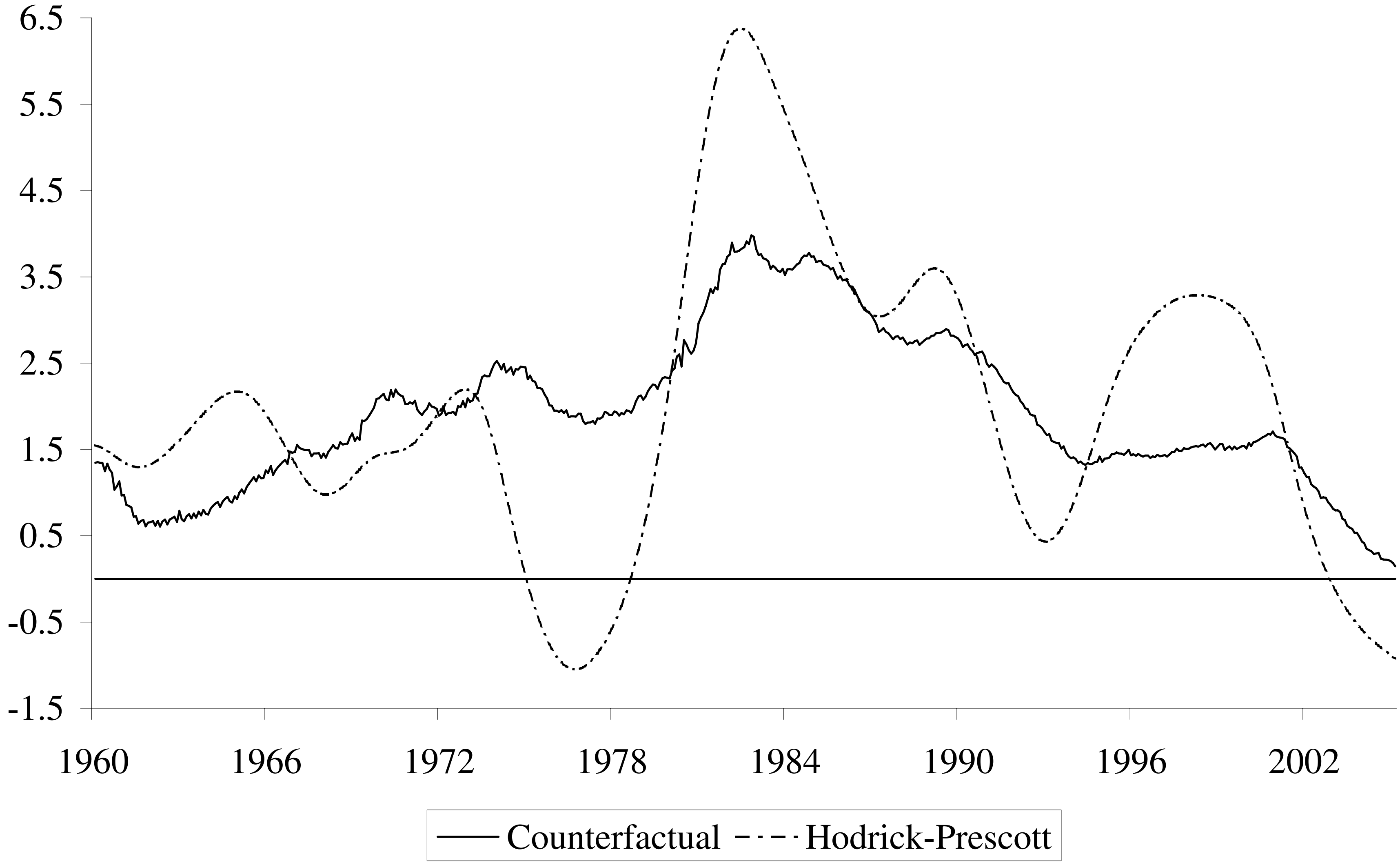


Figure 10: Counterfactual Real Federal Funds Rate and Hodrick-Prescott Trend



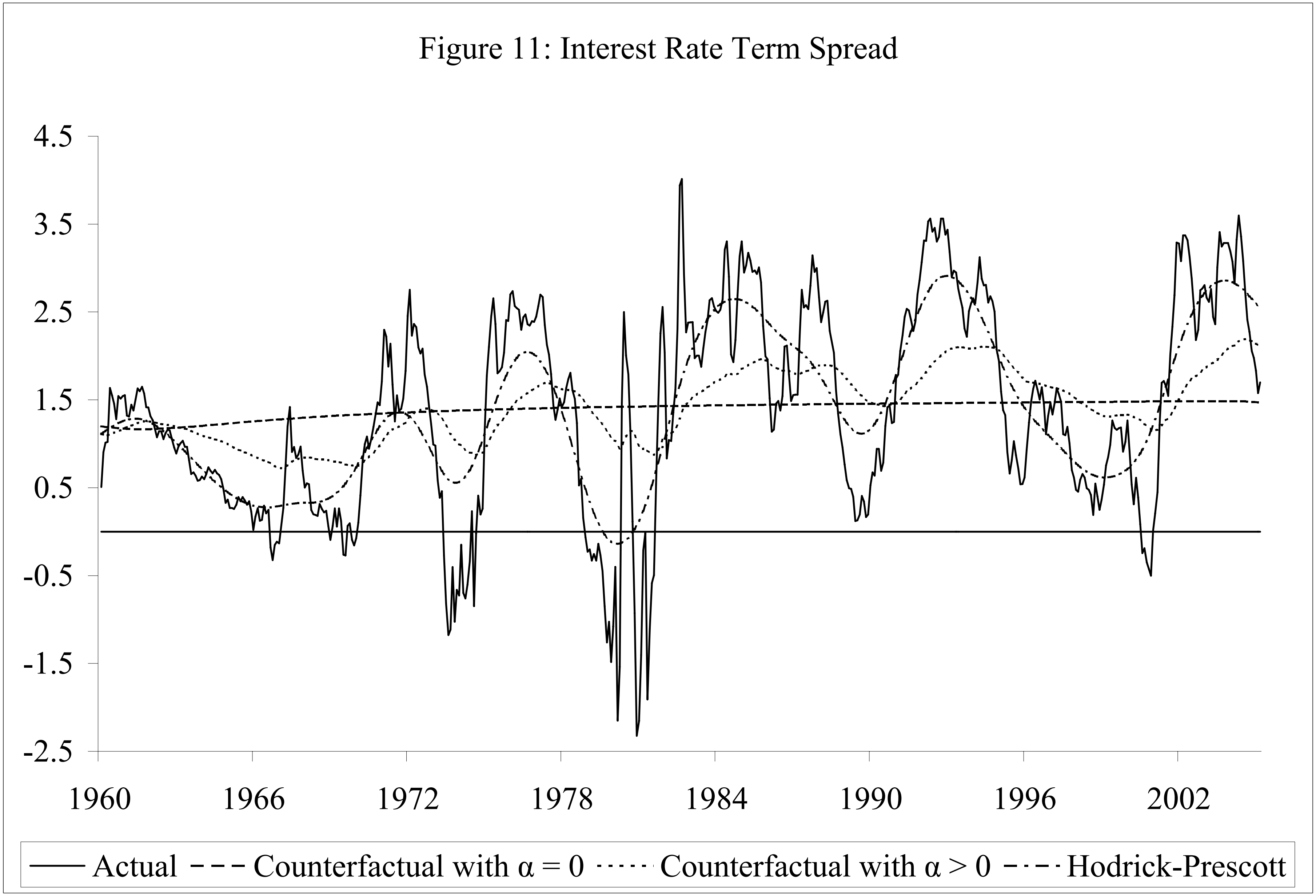


Figure 11: Interest Rate Term Spread
4.5
3.5
2.5
1.5
0.5
-0.5
-1.5
-2.5
1960
1966
1972
1978
1984
1990
1996
2002
Actual
Counterfactual with α = 0
Counterfactual with α > 0
Hodrick-Prescott

Figure 12: Smoothed Periodograms of Business Cycle Index and Cyclical Components of Data

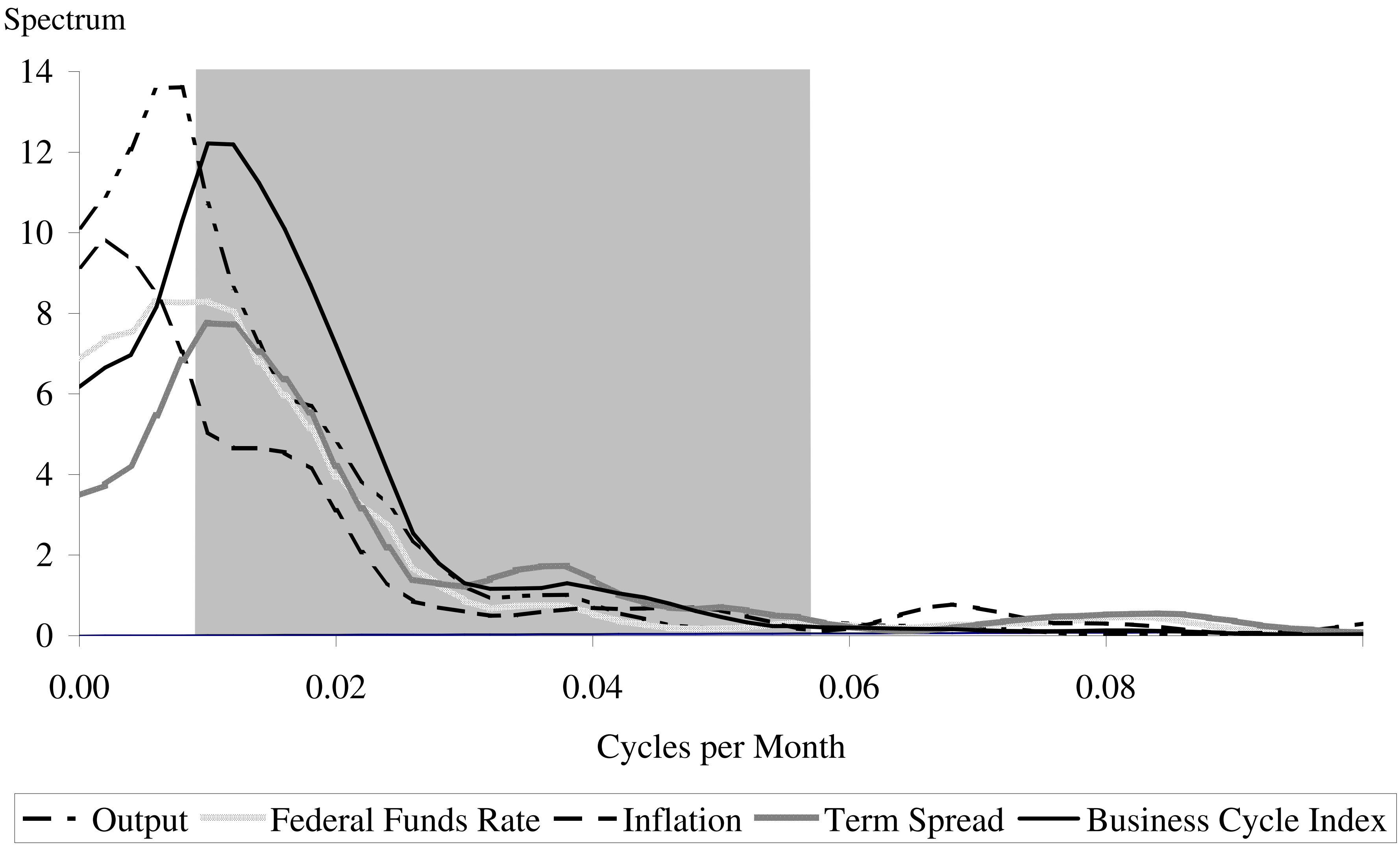


Table 1: Cyclical Behavior of Economy With Alternative Business Cycle Filters

Correlations with Business Cycle Index — Counterfactual Filter

	t-12	*t-9*	*t-6*	*t-3*	*t-1*	*t*	*t+1*	*t+3*	*t+6*	*t+9*	*t+12*
Business Cycle Index	0.490	0.656	0.809	0.934	0.989	1.000	0.989	0.934	0.809	0.656	0.490
Inflation	-0.254	-0.251	-0.238	-0.247	-0.198	-0.157	-0.111	-0.061	-0.007	0.064	0.136
Federal Funds Rate	-0.652	-0.647	-0.596	-0.510	-0.385	-0.312	-0.241	-0.115	0.017	0.135	0.227
Term Spread	0.649	0.617	0.524	0.376	0.214	0.129	0.050	-0.082	-0.212	-0.319	-0.393
Output	-0.327	-0.223	-0.097	0.042	0.156	0.214	0.270	0.362	0.445	0.489	0.482

Correlations with Business Cycle Index — Hodrick-Prescott Filter

	t-12	*t-9*	*t-6*	*t-3*	*t-1*	*t*	*t+1*	*t+3*	*t+6*	*t+9*	*t+12*
Business Cycle Index	0.490	0.656	0.809	0.934	0.989	1.000	0.989	0.934	0.809	0.656	0.490
Inflation	-0.094	-0.120	-0.148	-0.209	-0.181	-0.144	-0.104	-0.071	-0.051	-0.002	0.054
Federal Funds Rate	-0.408	-0.475	-0.487	-0.463	-0.332	-0.247	-0.165	-0.031	0.069	0.150	0.198
Term Spread	0.422	0.467	0.440	0.345	0.190	0.106	0.030	-0.085	-0.161	-0.213	-0.230
Output	-0.396	-0.336	-0.235	-0.091	0.067	0.153	0.227	0.342	0.430	0.472	0.445

Correlations with Business Cycle Index — Band Pass Filter

	t-12	*t-9*	*t-6*	*t-3*	*t-1*	*t*	*t+1*	*t+3*	*t+6*	*t+9*	*t+12*
Business Cycle Index	0.490	0.656	0.809	0.934	0.989	1.000	0.989	0.934	0.809	0.656	0.490
Inflation	-0.187	-0.177	-0.169	-0.200	-0.184	-0.160	-0.137	-0.133	-0.155	-0.161	-0.156
Federal Funds Rate	-0.291	-0.292	-0.298	-0.318	-0.292	-0.271	-0.255	-0.236	-0.253	-0.257	-0.250
Term Spread	0.424	0.376	0.343	0.324	0.249	0.206	0.167	0.115	0.101	0.054	-0.005
Output	-0.369	-0.321	-0.244	-0.145	-0.075	-0.040	-0.008	0.049	0.107	0.129	0.116